as a function of M for five different values of N are presented in Fig. 3.

For comparison, the fraction of the total energy emitted by S_0 and directly incident upon S_2, without reflection, is shown in Fig. 4.

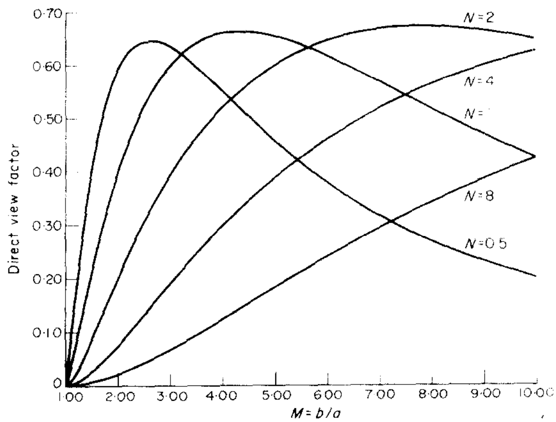

FIG. 4. Fraction of energy emitted by right fin and received directly by left fin without reflection.

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MULTIVALUED RELATIONS BETWEEN SURFACE CONDUCTION AND SURFACE TEMPERATURE IN A SATURATED POROUS MEDIA WITH PHASE CHANGE

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NOMENCLATURE

C_p, specific heat;
h, enthalpy;
h_{fg}, latent heat of vaporization;
k, thermal conductivity;
K, permeability;
L, length of porous section;
$\dot{m}$, mass flow rate per unit area $= \rho v$;
p, pressure;
Pe, Péclét number $= mC_pL/k = RePrL/d$;
q, heat flux per unit area;
S, interface position;
T, temperature;
x, distance coordinate.

Greek symbols
Δ, conduction–convection difference;
θ, dimensionless temperature;
μ, viscosity;
ν, kinematic viscosity $= \mu/\rho$;
ρ, convection–conduction ratio.

Dimensionless quantities
C, $\equiv C_{pL}/C_{pv}$;
F, $\equiv \dot{m}_L/\dot{m}_i$;
H, $\equiv h_{fg}(T^*)/[h_L(T^*) - h_L(T_R)]$;
R, $\equiv \nu_v/\nu_L$;
δ, $\equiv L\Delta/k\nu_{eff}(T^* - T_R)$;
κ, $\equiv k_{L_{eff}}/k_{v_{eff}}$;
ψ, $\equiv d\hat{q}_0/dS$.

Subscripts
- eff, effective property;
- con, conduction;
- i, initial value (before interface advances into medium);
- 0, value at $x = 0$;
- R, reservoir ($x = L$);
- f, fluid (liquid or vapor);
- L, liquid;
- V, vapor.

Superscripts
- *, saturation conditions;
- ^, dimensionless quantity.

INTRODUCTION

THE PROBLEM investigated here was first discussed in [1]. Liquid from a constant temperature reservoir flows into one end of a porous medium. At the opposite end, a constant temperature is imposed. For high enough values of the surface temperature, an evaporation front, separating a vapor region from a liquid region, propagates into the medium.

It is shown in [1] that for the constant temperature boundary condition one steady-state interface position can exist. However, for a prescribed constant heat flux at the boundary either one or three steady-state interface positions, depending on the physical parameters of the problem, can exist. Since the relation between surface temperature and interface position is single-valued [1], we are lead to the possibility that for a given heat flux, to which more than one interface position can be attributed, one may find more than one surface temperature. In other words, we expect that the relation between surface temperature and surface heat conduction is multivalued. Consequently, the governing equations of [1] are further investigated here and it is shown that indeed such a multivalued relation exists.

GOVERNING EQUATIONS

The model and physical assumptions of [1] are fully retained as well as the nomenclature and the method of normalization. For a detailed description the reader is referred to [1]. For brevity we refer freely to the steady-state formulation of [1] and repeat here the basic normalized equations:

$$\frac{d^2\theta_v}{dx^2} + \frac{\kappa Pe_i F(S)}{C}\frac{d\theta_v}{dx} = 0, \qquad 0 \leqslant x \leqslant S \tag{1}$$

$$\frac{d^2\theta_L}{dx^2} + Pe_i F(S)\frac{d\theta_L}{dx} = 0, \qquad S \leqslant x \leqslant 1. \tag{2}$$

The corresponding boundary conditions are

$$\text{at } x = 0: \theta_v = 0 \tag{3}$$

$$\text{at } x = 1: \theta_L = 0 \tag{4}$$

$$\text{at } x = S \begin{cases} \theta_v = \theta_L = 1 & (5) \\ \dfrac{d\theta_L}{dx} + \dfrac{\theta_0}{\kappa}\dfrac{d\theta}{dx} = HPe_iF(S). & (6) \end{cases}$$

Equations (1)–(5) yield the normalized temperature distributions:

$$\theta_v(x) = \frac{1 - \exp[-\kappa Pe_i F(S)x/C]}{1 - \exp[-\kappa Pe_i F(S)S/C]}, \quad 0 \leqslant x \leqslant S \tag{7}$$

$$\theta_L(x) = \frac{\exp[Pe_i(1-x)F(S)] - 1}{\exp[Pe_i(1-S)F(S)] - 1}, \quad S \leqslant x \leqslant 1. \tag{8}$$

The position of the interface, S, is given by the substitution of (7)–(8) into (6):

$$\frac{\theta_0}{C} = \left\{\exp\left[\frac{Pe_i\kappa SF(S)}{C}\right] - 1\right\} \times \left\{H + \frac{1}{1 - \exp[-Pe_i(1-S)F(S)]}\right\}, \tag{9}$$

where

$$F(S) = \frac{1}{RS + 1 - S}, \tag{10}$$

and the initial Péclét number, Pe_i, is based on the flow rate when the interface is at $x = 0$: $Pe_i = \dot{m}_i Cp_L L/k_{L_{eff}}$, and $\dot{m}_i = K(P_R - P_0)/Lv_v$.

CONDUCTION HEAT FLUX AT $x = 0$

From the definition of $\theta_0 \equiv (T_0 - T^*)/(T^* - T_R)$ and $\theta_v \equiv (T_0 - T_v)/(T_0 - T^*)$ we form the one dimensional heat flux:

$$q_{con} \equiv -k_{v_{eff}}\left|\frac{dT_v}{dx}\right|_{x=0} = +\frac{k_{v_{eff}}}{L}\theta_0(T^* - T_R)\left|\frac{d\theta_v}{dx}\right|_{x=0},$$

hence, the normalized conduction at the surface can be written as:

$$\hat{q}_0 \equiv \frac{L}{k_{v_{eff}}(T^* - T_R)}q_0 = \theta_0\left(\frac{d\theta_v}{dx}\right)_{x=0}. \tag{11}$$

Since

$$\left(\frac{d\theta_v}{dx}\right)_{x=0} = \frac{\kappa Pe_i F(S)/C}{1 - \exp[-Pe_i F(S)S/C]},$$

$$\hat{q}_0 = \frac{\theta_0 \kappa Pe_i F(S)/C}{1 - \exp[-\kappa Pe_i F(S)S/C]}. \tag{12}$$

Limit of $Pe_i \to 0$

From [1] we have

$$S \sim \frac{\theta_0/\kappa}{1 + \theta_0/\kappa} \tag{13}$$

as $Pe_i \to 0$. Substituting (13) into (12) and making use of (11) we obtain:

$$\hat{q}_0 \sim (\theta_0 + \kappa) \qquad \text{for } x = 0 \neq S \tag{14}$$

$$\hat{q}_0 \sim \kappa \qquad \text{for } x = 0 = S. \tag{15}$$

Limit of $Pe_i \to \infty$

From the limit of $Pe_i \to \infty$, we retain [1] the expression

$$S \sim \frac{C}{\kappa Pe_i} \ln\left[1 + \frac{\theta_0}{C(H+1)}\right]. \qquad Pe_i \to \infty. \tag{16}$$

In this limit

$$F(S) \sim \frac{\kappa Pe_i}{C(R-1)\ln\{1 + [\theta_0/C(H+1)]\} + \kappa Pe_i},$$

which after substitution in (12) and rearranging yields

$$\hat{q}_0 \sim \frac{\kappa Pe_i}{C}[\theta_0 + C(H+1)], \qquad Pe_i \to \infty. \tag{17}$$

SURFACE HEAT FLUX

To determine the energy convected out with the vapor at $x = 0$, use is made of [1]:

$$\dot{m}_v = \dot{m}_L = \dot{m}_i F(S) = F(S)\frac{Pe_i k_{L_{eff}}}{Cp_L L};$$

$$\dot{m}_v h_v(T_0) = \frac{F(S)Pe_i k_{L_{eff}} h_v(T_0)}{Cp_L L},$$

and using the approximation $h_v(T_0) \cong Cp_v T_0$ then

$$\dot{m}_v h_v(T_0) = \frac{F(S)Pe_i k_{L_{eff}} T_0}{CL} = \frac{F(S)Pe_i \kappa k_{v_{eff}} T_0}{CL}. \tag{18}$$

Forming now the ratio, ρ, of convection flux to conduction flux at the surface, (12) and (18) yield

$$\rho \equiv \left\{1 - \exp\left[\frac{-\kappa Pe_i F(S)S}{C}\right]\right\}\left[1 + \frac{T^*}{\theta_0(T^* - T_R)}\right]. \tag{19}$$

Assuming now that the heat input by conduction (for the case of an imposed surface temperature) is larger than the energy convected out with the vapor, the difference becomes

$$\Delta \equiv \frac{k_{v_{eff}}}{L}\theta_0(T^* - T_R)\left(\frac{d\theta_v}{dx}\right)_{x=0} - \frac{F(S)Pe_i \kappa k_{v_{eff}} T_0}{CL}.$$

Defining

$$\delta \equiv \frac{L\Delta}{k_{v_{eff}}(T^* - T_R)} \tag{20}$$

we obtain

$$\delta = \frac{F(S)Pe_i\kappa}{C}\left\{\frac{\theta_0}{1 - \exp[-\kappa Pe_i F(S)S/C]} - \left(\theta_0 + \frac{T^*}{T^* - T_R}\right)\right\}. \tag{21}$$

Limit of $Pe_i \to 0$.

Using (13) we obtain:

$$\rho \sim \frac{\kappa Pe_i F(S)S}{C}\left[1 + \frac{T^*}{\theta_0(T^* - T_R)}\right] \to 0, \tag{22}$$

$$\delta \sim \frac{\theta_0}{S}. \tag{23}$$

If the Péclét number is to be interpreted as providing a measure of the relative magnitude of heat transfer by convection to heat transfer by conduction, we indeed expect $\rho \to 0$. Equation (23) can be written, with (13) as $\delta \sim (\theta_0 + k)$, $Pe_i \to 0$, which is consistent with (14) since in this limit conduction dominates.

Limit of $Pe_i \to \infty$

As the Péclét number increases convection becomes more and more dominant and for $Pe_i \to \infty$ the position of the interface moves to the origin minimizing the effect of conduction. This deduction is consistent with (16). In this limit, of $Pe_i \to \infty$ and $S \to 0$, the interface function $F(S) \to 1$ and thus the exponent in the expression for ρ yields

$$\frac{\kappa Pe_i F(S)S}{C} \to \ln\left[1 + \frac{\theta_0}{C(H+1)}\right].$$

Finally,

$$\rho \sim \frac{1}{\theta_0 + C(H+1)}\left[\theta_0 + \frac{T^*}{T^* - T_R}\right], \qquad Pe_i \to \infty. \tag{24}$$

$$\delta \sim \frac{Pe_i\kappa}{C}\left\{C(H+1) - \frac{T^*}{T^* - T_R}\right\}, \qquad Pe_i \to \infty. \tag{25}$$

ZERO NET FLUX

Consider the case in which the heat conducted into the porous medium equals the energy flux carried out by convection. Setting (21) to equal zero,

$$\frac{T^*}{T_0} = \exp\left[\frac{-\kappa Pe_i F(S)S}{C}\right]. \tag{26}$$

Consistent with the condition that for finite values of Pe_i, $S = 0$ when $T^* = T_0$. To retain the condition of zero net heat flux at the surface, $x = 0$, for the case $S = 1$, (26) yields

$$T_0 = T^* \exp(\kappa Pe_i/RC). \tag{27}$$

One may compute now the heat flux due to conduction for the special case of the evaporation front being located at the origin $x = S = 0$. This is done by considering $\theta_0 = \theta_0(S)$

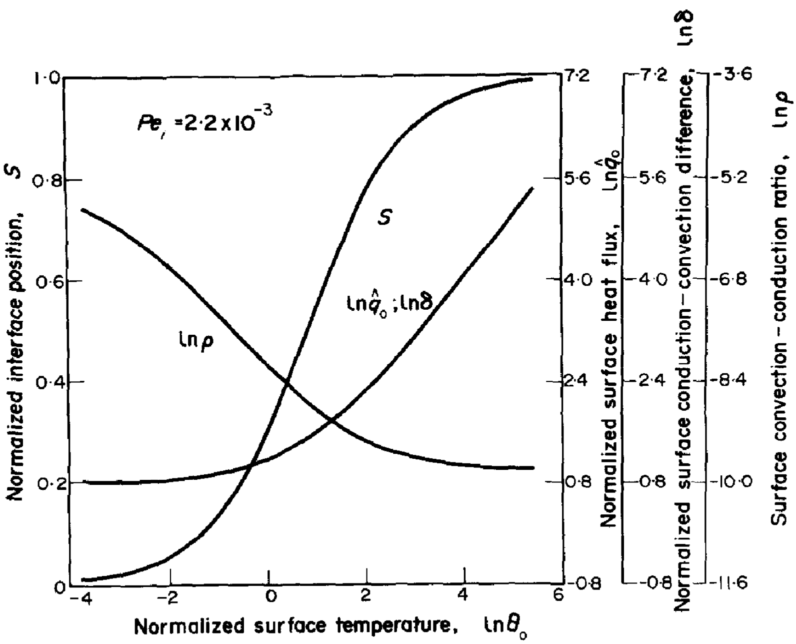

FIG. 1. Normalized interface position, S, normalized surface conduction, $\hat{q}_0$, conduction–convection difference, δ, and convection–conduction ratio, ρ, as a function of the normalized surface temperature θ_0. $Pe_i = 2{\cdot}2\times10^{-3}$.

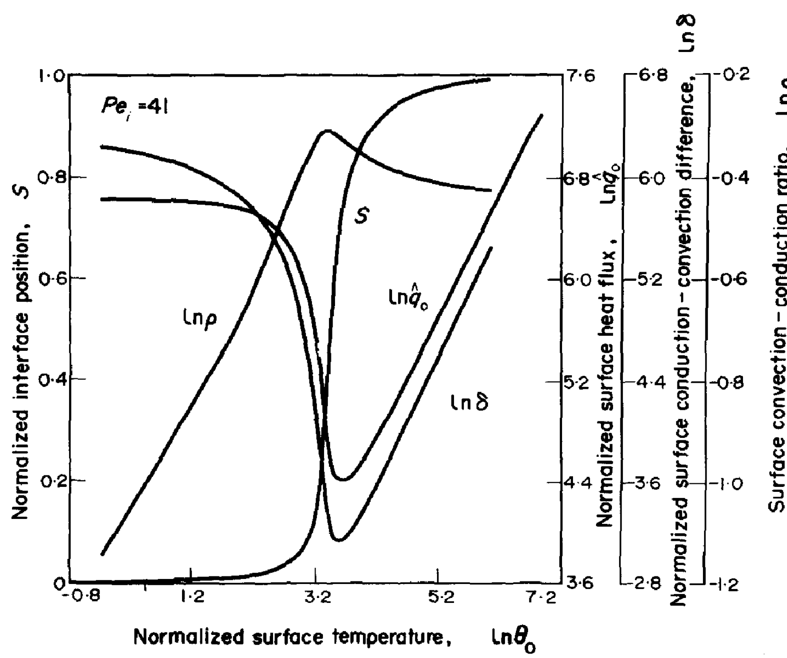

FIG. 2. Normalized interface position, S, normalized surface conduction, $\hat{q}_0$, conduction–convection difference, δ, and convection–conduction ratio, ρ, as a function of the normalized surface temperature θ_0. $Pe_i = 41$.

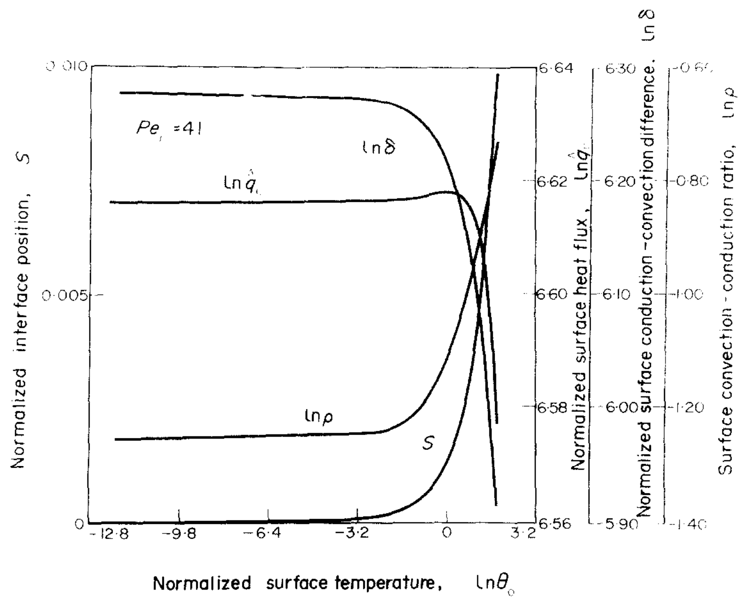

FIG. 3. Normalized interface position, S, normalized surface conduction, $\hat{q}_0$, conduction–convection difference, δ, and convection–conduction ratio, ρ as a function of the normalized surface temperature θ_0. $Pe_i = 41$.

from (9) and substituting it in the expression for $\hat{q}_0$ as given by (12):

$$\lim \hat{q}_0 = \kappa Pe_i \left\{ H + \frac{1}{1 - e^{-Pe_i}} \right\}, \quad x = S = 0. \quad (28)$$

In the limit of $Pe_i \to 0$ and $Pe_i \to \infty$ this result is consistent with (15) and (17) respectively, and with the idea of minimum heat flux, q^*, as given in [1].

NUMERICAL COMPUTATIONS AND RESULTS

Equations (9), (10), (12), (19) and (21) are solved numerically for the same physical parameters as in [1]: $R = 45$, $H = 7{\cdot}1$, $C = 1{\cdot}96$, $\kappa = 2{\cdot}24$, $T_R = 25°C$, $T^* = 100°C$. The plotted results indicate clearly the multivalued effect. In the limit of a vanishingly small Péclét number, Fig. 1, the curves for $\hat{q}_0$ and for δ coincide. This is due to the fact that $\hat{q}_0 \to \theta_0/S$ and $\delta \to \theta_0/S$ for $Pe_i \to 0$. As the Péclét number increases, the curves become different.

For all finite values of Pe_i two differently imposed surface temperatures, θ_0, giving rise to two distinctly different interface positions, yield the same conduction flux at the surface.

For a given Péclét number the increasing surface temperature, and hence the continuous penetration of the interface position, are associated with a strong decrease in the conduction flux, followed by an immediate increase. This response in the steady-state formulation may give rise to some non-physical situations as a result of instabilities that are suggested clearly in Fig. 2. Consider moving along the ln $\hat{q}_0$ curve in the direction of decreasing values of θ_0. Down to a certain value of $\hat{q}_0$ the relation with θ_0 is single valued. Below this value the same value of $\hat{q}_0$ can be maintained with two or even three different surface temperatures. This is due to the fact that in Fig. 2 $\hat{q}_0$ has a maximum that occurs at a surface temperature, θ_0, slightly larger than unity. The scale of Fig. 2 does not allow proper presentation and indicates only the possibility of two surface temperatures for the same $\hat{q}_0$. In Fig. 3 the scale of S, $\hat{q}_0$, δ, and ρ are expanded to show the behaviour near the local maximum and the possibility of three surface temperatures corresponding to the same conduction flux is indicated.

The curves for $\hat{q}_0$ show also a minimum. This minimum occurs at a surface temperature associated with a value of $S \neq 0$. In all cases the minimum value of $\hat{q}_0$ is lower than the minimum heat conduction, q^*, required for surface evaporation (28). This difficulty may be resolved by a careful stability analysis.

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